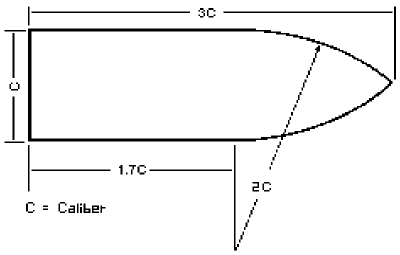
A Very Simple Guide To the Ballistic Coefficient
(with particular reference to airgun ballistics)

What is it?
The Ballistic Coefficient (BC) is a crude - but useable - calculated value that describes the aerodynamic efficiency of a projectile in flight. Higher BC values suggest better aerodynamic efficiency than do lower ones. A pellet with a higher BC value will retain more of it's muzzle velocity (and hence, energy) downrange. Since more of the initial (muzzle) velocity is retained, the trajectory is 'flatter' and the time-to-target is reduced – resulting in less wind-drift.
A High BC value is, therefore, *A Good Thing*.

How is it defined?
The classical definition refers back to a projectile and model originally developed by Krupp and analysed by Zabudski Mayevski (around 1881). Mayevski's analytical expressions were later developed and tabulated by Col. James Ingalls and published in 1893.



The 'standard' Ingalls projectile had a mass of one Pound, a diameter or one Inch and was 3 Inches long. It was flat based and had an ogival nose radius of 2 Inches.

This 'standard' projectile has a **defined** BC value of 1

Ballistic Coefficient is **defined** as:

$$\mathbf{BC = SD/FF = M/(d^2*FF)} \qquad \mathbf{(0)}$$

Where:
BC = Ballistic Coefficient
M = Projectile Mass (Lb)
d = Projectile Diameter (Inch)
SD = Sectional Density = Mass/d^2
FF = Form Factor

The standard projectile has SD = 1/(1 * 1) = 1 (Lbf/in^2). Since the BC value is defined as unity, the Form Factor must be equal to SD/BC = 1/1 = 1. Note that it is usual to **define** the Form Factor to have the same units as Sectional Density so that the BC value is dimensionless.

This definition is used to compare the performance of other projectiles - that may differ in their mass, diameter or Form Factor – to that of the standard projectile.

While of some interest, the BC definition above is not of much practical use. However, the generalities do give us an insight. For example, if you want to **increase** the BC value you could:

1) **Increase** the pellet mass (for the same diameter and Form Factor) – this is a viable solution depending of the availability of suitable pellets – or,

2) **Reduce** the Form Factor (for the same mass and diameter) – may be an option depending on the available alternatives. Practical experience suggests that the modern round-nosed pellet has the lowest Form Factor available. Flat-nosed, H-Point and pointed pellets are generally inferior (in Form Factor and BC terms) although, of course, there may be other valid reasons for using them.

How is it calculated?

One of the most useful expressions for calculating BC is:

$$\mathbf{BC = (R_1 - R_0)/(\log_e(V_0/V_1)*8000)} \qquad \mathbf{(1)}$$

Where:
BC = Ballistic Coefficient
$\mathbf{R_0}$ = Near Range (Yard)
$\mathbf{R_1}$ = Far Range (Yard)
$\mathbf{V_0}$ = Velocity at $\mathbf{R_0}$ (Ft/s^2), and
$\mathbf{V_1}$ = Velocity at $\mathbf{R_1}$ (Ft/s^2)

You'll need two chronographs of known accuracy and an accurate means of measuring distance – a surveyor's tape perhaps. In most cases, the first chronograph would be positioned at (or close to) the muzzle so the R_0 term will be zero and the V_0 term would be equal to the Muzzle Velocity. Very short ranges are to be avoided (i.e., ($R_1 - R_0$) less than ~ 20 Yards) since the velocity difference – compared to the chronograph's accuracy – may be too small to yield an accurate result. On the other hand, long ranges (i.e., ($R_1 - R_0$) greater than ~ 80 Yards) may be suspect due to the onset of instability. See 'Other considerations' below.
Because of the degree of uncertainty involved (chronograph accuracy, pellet variations, etc.), you should also be prepared to repeat the test shot several times – ten to twenty times – and apply the expression using the average velocities.
The above expression is correct for ambient conditions of Normal Temperature and Pressure (NTP). If the ambient conditions are not 'normal', then they can be appropriately compensated – see 'Other Considerations' below.

So what do I do with it now?

Having established a BC value, other interesting values may now be calculated:

Downrange (retained) Velocity:

$$\mathbf{V_1 = V_0/(Exp(R_1/(8000*BC)))\ \ldots\ Ft/s} \qquad \mathbf{(2)}$$

where:
$\mathbf{V_1}$ = Velocity at $\mathbf{R_1}$ (Ft/s)
$\mathbf{V_0}$ = Initial (muzzle) Velocity (Ft/s)
$\mathbf{R_1}$ = Range (Yard), and
BC = Ballistic Coefficient

Time to Target:

$$\mathbf{T = 24000*BC*(Exp(R_1/(8000*BC))-1)/V_0 \quad ... \ Seconds \qquad (3)}$$

where:
T = Transit time between muzzle and target (sec.)
$\mathbf{V_0}$ = Muzzle Velocity (Ft/s)
$\mathbf{R_1}$ = Distance between muzzle and target (Yard)
BC = Ballistic Coefficient

So, being able to calculate $\mathbf{V_1}$ (the velocity at some downrange target) and **T** (the time taken to reach the downrange target) we can now calculate:

Downrange (retained) Energy:

$$\mathbf{E_1 = M*V_1^2/K \qquad ... \ Ft\text{-}Lbf}$$

where:
$\mathbf{E_1}$ = Energy (Ft-Lbf)
M = Pellet Mass (Grain)
$\mathbf{V_1}$ = Pellet Velocity (Ft/s) … from expression **(2)** above
K = Konstant = 7000 * 2 * 32.16 ~ = 450240

Of course, if $\mathbf{V_1}$ = $\mathbf{V_0}$ (the Muzzle Velocity) then $\mathbf{E_1}$ = Muzzle Energy.

Wind Deflection:

$$\mathbf{D = 12*W*sin(A)*(T-(3*R_1/V_0)) \quad ... \ Inch}$$

where :
D = Wind Deflection (Inch)
W = Wind Velocity (Ft/s)
A = Angle between Wind Velocity vector and the trajectory
T = Transit time between muzzle and target (sec.)
… from expression **(3)** above
$\mathbf{V_0}$ = Muzzle Velocity (Ft/s)
$\mathbf{R_1}$ = Distance between muzzle and target (Yard)

Note that the worst case (i.e., maximum lateral deflection) is when A = 90^o so that sin(A) = 1 and that the ($3*R_1/ V_0$) term is equal to the time-of-flight in a vacuum, i.e., constant velocity with no drag.

Other considerations …

Ah, would it be that simple …. alas, no. Airgun users have, to a large extent, been handed the mucky end of the stick – as far as the physics of ballistics is concerned.
With such low BC values and low velocities, everything becomes critical.

Measurement Problems

The accuracy of an BC calculation (using the (1) expression above) can only be as good as the accuracy of the velocity and range measurements. Measuring the range is not usually too much of a problem; an error of say ± 3 Inches in 50 Yards is only ~ 0.17% and would equate to a similar error in the calculated BC value.
An error in the velocity measurement is much more serious. Let's assume that the basic accuracy of a matched pair of chronographs is ± 0.5% of the reading and that the display rounds the reading to the nearest whole digit. We'll assume that the repeatability is perfect – unlikely, but bear with me on this. If the two chronographs are 20 Yards apart and the velocity indications are 800 and 740 Ft/s respectively then what is the BC value?

On the face of it (and applying expression (1) above) :

BC = $20/(\log_e(800/740) * 8000)$ = **~0.0320**

… but …
what if V_0 was really 800 * (1.005) = 804 Ft/s
and V_1 was really 740 * 0.995 = 735 Ft/s (rounded down from 735.3 Ft/s) ?
then BC = $20/(\log_e(804/735) * 8000)$ = **~0.0278**

… or …
if V_0 was really 800 * (0.995) = 796 Ft/s (rounded up from 795.5 Ft/s)
and V_1 was really 740 * 1.005 = 744 Ft/s (rounded down from 744.2 Ft/s) ?
then BC = $20/(\log_e(796/744) * 8000)$ = **~0.0370**

That's a simple systematic error of ~±15% - **thirty times** the assumed accuracy of the chronograph. Add the effects of temperature, humidity, repeatability, differential calibration and offset drift, etc., and the extent of the problem is obvious.
Spending more on a pair of chronographs may reduce the errors slightly but there will always be the classic struggle to keep two-of-anything properly calibrated and synchronised. A Doppler Radar chronograph would probably be the ultimate solution but the cost of such items makes them affordable only to arms manufacturers and governments. Realistically, who's going to spend £60,000 + on equipment to test airgun pellets?

Repeatability Problems

The BC value can vary between different barrels (up to ~±15%), in the same batch (up to ±5%) or with environmental conditions. The problem is that the effect on the trajectory of ±15% on a BC value of (say) 0.03 is vastly greater than on a small-arms round having a BC of (say) 0.3 ±15%
BC increases by about 3% per 1000 Feet of altitude, and by about 0.3% per ^{o}C of ambient temperature. The effects of relative humidity can be observed but, since they are very small compared to the other environmental variables, they are generally ignored.

Stability Problems

Most modern airgun pellets, shot from a normal airgun barrel (with the normal 1:~16 twist rate) at normal airgun (sub-sonic) velocities, are probably barely stable at normal airgun ranges – most of the stability originating from the immense drag generated by the skirt. However, if any of the parameters are outside of the 'normal' airgun spectrum then the effects of instability may be observed as a changing BC value. Depending on the particular set of parameters, the BC value may start off low and progressively improve or, in most cases, diminish with velocity/range as the pellet becomes progressively more unstable.

How does instability affect the BC value? In two fundamental ways. With the onset of instability, the pellet 'wobbles' on it's axis and, in so doing, it presents an increasingly greater area to the air-stream. Since BC is proportional to the Sectional Density and Sectional Density is inversely proportional to the cross-sectional area, the BC value is reduced. As the 'wobble' progresses, the Form Factor is also increasing and, since the BC value is inversely proportional to the Form Factor, the Ballistic Coefficient is again diminished.

The point of all this is that the range over which the BC value is measured must be given due consideration or erroneous BC values may result.

Velocity Problems

It is normally assumed that the BC value of an airgun pellet is constant as long as the velocities are less than ~ 950 Ft/s. The bad news is that the BC value is probably not constant through any sub-sonic velocity range. The good news is that the indications suggest that the deviation is approximately linear and the slope small enough to be swamped by, fore instance, the effects of the onset of instability or the statistical uncertainty associated with the observations/calculations.

Many Thanks

The closed-solution expressions marked (1), (2) and (3) above are believed to be the original work of Steve Woodward (a.k.a. Steve_in_NC, pneuguy) as are other expressions used in various ChairGun programs and other ballistic software.
Many thanks for making it so easy.